A Constrained Optimization Perspective on Joint Spatial Resolution and Dynamic Range Enhancement

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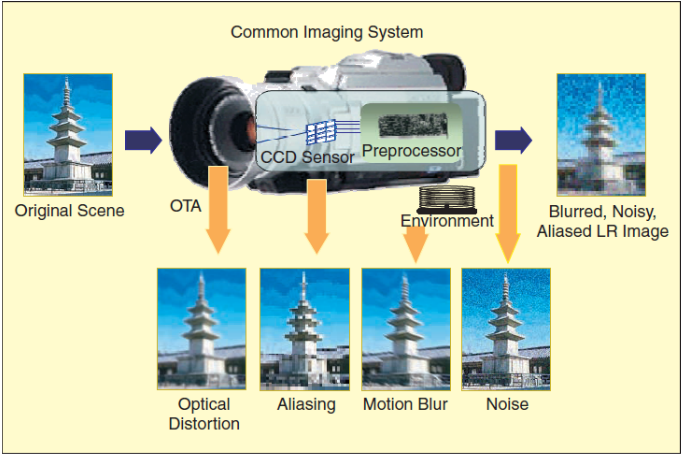
Digital image acquisition system[1]



[1] Park et al., IEEE Signal Processing Magazine, 2003

Image super-resolution (SR)

- Classical inverse problem in image processing[2] (1984-present)
- Estimate the unknown high resolution (hi-res) image from low resolution (lo-res) image(s)
  - Optical SR: single lo-res image, frequency extrapolation beyond diffraction limit
  - Digital SR: multiple lo-res captures, frequency extrapolation beyond imaging system bandwidth
- Complementary yet correlated information in multiple images
- Lo-res image cues: sub-pixel shifts, zoom, blur
- Applications: military surveillance, medical imaging, synthetic aperture radar, thermal imaging, consumer electronics, etc.

[2] Tsai and Huang, Advances in Computer Vision and Image Processing, 1984

Model of the forward imaging process

$$\mathbf{y}_k = \mathbf{DBT}(\boldsymbol{\theta}_k)\mathbf{x} + \mathbf{n}_k, \quad 1 \leq k \leq K$$

where

- $\mathbf{x} \in \mathbb{R}^n$ is the unknown hi-res image
- $\mathbf{y}_k \in \mathbb{R}^m$ $(m < n)$ represents the k-th lo-res image
- $\mathbf{T}(\boldsymbol{\theta}_k) \in \mathbb{R}^{n \times n}$ is the k-th geometric warping matrix
  - $\boldsymbol{\theta}_k$ obtained from projective homography matrix[3]
- $\mathbf{B} \in \mathbb{R}^{n \times n}$ describes camera optical blur
- $\mathbf{D} \in \mathbb{R}^{m \times n}$ is a downsampling matrix of $1s$ and $0s$
- $\mathbf{n}_k \in \mathbb{R}^m$ is the noise vector that corrupts $\mathbf{y}_k$.

[3] Mann and Picard, IEEE Trans. Image Processing, 1997

Cost function formulation

$$\mathcal{C} = \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{DBT}(\boldsymbol{\theta}_k)\mathbf{x}\|_p + \lambda\rho(\mathbf{x}), p \geq 1$$

$$(\hat{\mathbf{x}}, \hat{\boldsymbol{\theta}}) = \arg\min_{\mathbf{x},\boldsymbol{\theta}} \mathcal{C}$$

Error norm minimization: dual of MAP estimation under a noise model.

- $\rho(\mathbf{x})$: regularization $\rightarrow$ stable solution
- Analogy to prior in MAP estimation

$$p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$$

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} p(\mathbf{y}|\mathbf{x}) \boxed{p(\mathbf{x})} \rightarrow \text{Prior}$$

- Example: $\rho(\mathbf{x}) = \|\mathbf{A}\mathbf{x}\|_2^2 \Leftrightarrow$ zero-mean Gaussian prior
- l_p norm: commonly $p = 1$ (Laplacian noise; robustness to outliers) or $p = 2$ (Gaussian).

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Early approaches to SR

1. Estimation of registration parameters $\boldsymbol{\theta}_k$ and hi-res image sequentially

2. Cost function minimization under:
   - different norms[4]
   - image prior/regularization models[5]

3. Joint MAP estimation of geometric registration parameters and hi-res image [5,6]

[4] Farsiu et. al., IEEE Trans. Image Processing, 2004

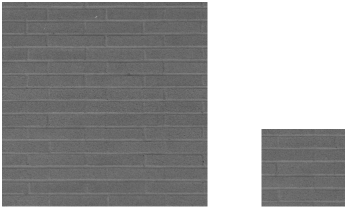
[5] Pickup et. al., EURASIP 2007

[6] Hardie et. al., IEEE Trans. Image Processing, 1997

Challenges in SR

Camera imaging model well-understood, but issues remain:

1. Tractability
   - Not jointly convex in $\mathbf{x}, \boldsymbol{\theta}$
   - Convexity in $\mathbf{x}$ (for fixed $\boldsymbol{\theta}$) well-known; not necessarily convex in $\boldsymbol{\theta}$



(a) (b)

Figure: (a) Patch of brick wall, (b) Sub-patch. Registration does not give a unique corresponding sub-patch in the original image.

   - Computational complexity
2. Faithfulness of resulting solutions to real-world constraints.

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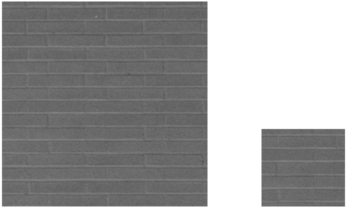

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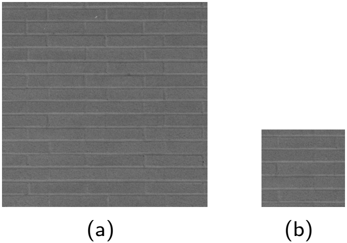

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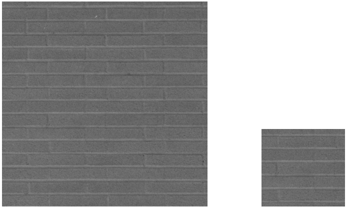

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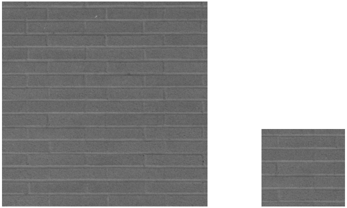

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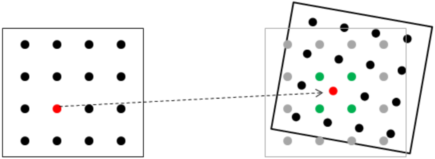
Contributions of our work

1. Separable convexity via transformation of variables[7] $\mathbf{f}_k : \boldsymbol{\theta}_k \mapsto \mathbf{T}(\boldsymbol{\theta}_k)$

   $\boldsymbol{\theta}$: change in pixel coordinates, $\mathbf{T}$: pixel intensity mapping.



$$\mathcal{C}(\mathbf{x}, \{\mathbf{T}_k\}, \mathbf{B}) = \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{D}\mathbf{B}\mathbf{T}_k\mathbf{x}\|_p + \lambda\rho(\mathbf{x}).$$

2. Formulation of elegant and physically meaningful convex constraints.

Why convexity?

- Convergence guarantee to minima
- Robustness to initialization values.

[7] Hindi, American Control Conference, 2004

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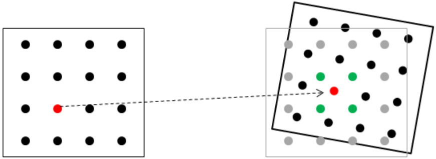

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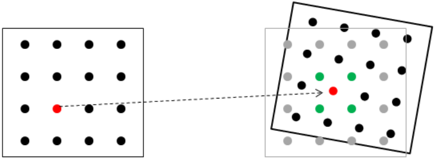

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- Separable convexity of $\mathcal{C}$ in $\mathbf{x}$ well-known.
- Convexity in $\mathbf{B}$ for fixed $\mathbf{x}$ and $\{\mathbf{T}_k\}$ follows from triangle inequality:

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& \mathcal{C}(\alpha \mathbf{B}_1 + (1-\alpha)\mathbf{B}_2) \\
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- Separable convexity in $\mathbf{T}_k$ shown similarly.
- Separable convexity holds for any l_p norm, $p \geq 1$.

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Formulation of convex constraints

- Ensure that imaging parameters correspond to real-world imaging physics; lead to physically meaningful images
- Non-negative pixel values of hi-res and lo-res images

$$\begin{aligned} \mathbf{0} &\leq \mathbf{x} \leq \mathbf{1} \\ \mathbf{0} &\leq \mathbf{DBT}_k\mathbf{x} \leq \mathbf{1}, \quad 1 \leq k \leq K \end{aligned}$$

- $\mathbf{T}_k$: interpolation matrix, $\mathbf{B}$: filtering with a local spatial kernel; each row should sum to 1

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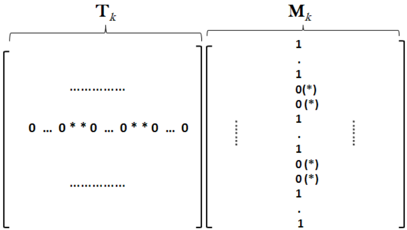

Figure: The $*$ in $\mathbf{T}_k$ indicate the non-zero values in every row. The corresponding locations in the column of $\mathbf{M}_k$ have zero entries, and all other entries are 1. ($\mathbf{M}_k = [\mathbf{m}_{k,1}\ \mathbf{m}_{k,2}\ \ \ldots \mathbf{m}_{k,n}]$.)

- Candidate set of non-zero entries in each row of $\mathbf{T}_k$ known:

$$\mathbf{t}_{k,i}^T \mathbf{m}_{k,i} = 0, \quad 1 \leq i \leq n, \quad 1 \leq k \leq K$$

  where $\mathbf{t}_{k,i} \in \mathbb{R}^n$ is the i-th row of $\mathbf{T}_k$, and $\mathbf{m}_{k,i} \in \mathbb{R}^n$ is the corresponding membership vector.

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- Constraints also exhibit separable convexity.

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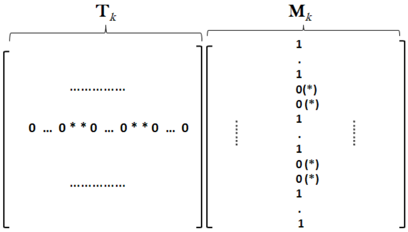

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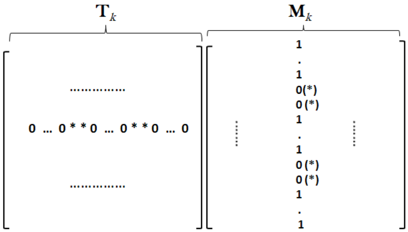

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Optimization problem

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\text{minimize} \quad & \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{DBT}_k\mathbf{x}\|_p + \lambda\rho(\mathbf{x}) \\
\text{subject to} \quad & \mathbf{0} \leq \mathbf{x} \leq \mathbf{1} \\
& \mathbf{0} \leq \mathbf{DBT}_k\mathbf{x} \leq \mathbf{1}, \quad 1 \leq k \leq K \\
& \mathbf{T}_k.\mathbf{1} = \mathbf{1}, \quad 1 \leq k \leq K \\
& \mathbf{B}.\mathbf{1} = \mathbf{1} \\
& \mathbf{t}_{k,i}^T \mathbf{m}_{k,i} = 0, \quad 1 \leq i \leq n, \quad 1 \leq k \leq K \\
& \mathbf{b}_i^T \mathbf{e}_i = 0, \quad 1 \leq i \leq n
\end{aligned}
$$

.

Is the camera imaging model complete?

- Assumption: Identical illumination conditions during lo-res capture
- Not valid in general
  - Variation in natural lighting, shadows, light sources in scene
  - Camera parameters - exposure time, aperture size, white balancing
- Incorporate photometric registration into model[8]
- Affine model: brightness gain and offset parameters

$$\mathcal{C}(\mathbf{x}, \{\mathbf{T}_k\}, \mathbf{B}, \boldsymbol{\lambda}) = \sum_{k=1}^{K} \|\mathbf{y}_k - \lambda_{\alpha_k}\mathbf{D}\mathbf{B}\mathbf{T}_k\mathbf{x} - \lambda_{\beta_k}\mathbf{1}\|_p + \gamma\rho(\mathbf{x})$$

- $\boldsymbol{\lambda} := [\lambda_{\alpha_1}, \lambda_{\beta_1}, \ldots, \lambda_{\alpha_K}, \lambda_{\beta_K}]^T$, $\mathbf{1} \in \mathbb{R}^m$: vector of all 1's
- Separable convexity property of $\mathcal{C}$ still holds!

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Final optimization problem

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} \|\mathbf{y}_k - \lambda_{\alpha_k}\mathbf{DBT}_k\mathbf{x} - \lambda_{\beta_k}\mathbf{1}\|_p + \gamma\rho(\mathbf{x}) \\
\text{subject to} \quad & \mathbf{0} \le \mathbf{x} \le \mathbf{1} \\
& \mathbf{0} \le \lambda_{\alpha_k}\mathbf{DBT}_k\mathbf{x} + \lambda_{\beta_k}\mathbf{1} \le \mathbf{1}, \quad 1 \le k \le K \\
& \lambda_{\alpha_k} > 0, \quad 1 \le k \le K \\
& \mathbf{T}_k.\mathbf{1} = \mathbf{1}, \quad 1 \le k \le K \\
& \mathbf{B}.\mathbf{1} = \mathbf{1} \\
& \mathbf{t}_{k,i}^T\mathbf{m}_{k,i} = 0, \quad 1 \le i \le n, \quad 1 \le k \le K \\
& \mathbf{b}_i^T\mathbf{e}_i = 0, \quad 1 \le i \le n
\end{aligned}$$

Constrained Alternating Convex Optimization algorithm

Algorithm 1 CACO

1: For initial estimates $\mathbf{x}_0$, $\{\mathbf{T}_k\}_0$, $\mathbf{B}_0$ and $\boldsymbol{\lambda}_0$ (available from state-of-the-art), optimize $\{\mathbf{T}_k\}$ as:

$$\begin{array}{rl} \{\mathbf{T}_k\}^* = & \arg\min\limits_{\{\mathbf{T}_k\}} \sum_{k=1}^{K} \|\mathbf{y}_k - \lambda_{\alpha_{k,0}} \mathbf{D}\mathbf{B}_0\mathbf{T}_k\mathbf{x}_0 - \lambda_{\beta_{k,0}}\mathbf{1}\|_p \\ \text{subject to} & 0 \le \lambda_{\alpha_{k,0}} \mathbf{D}\mathbf{B}_0\mathbf{T}_k\mathbf{x}_0 + \lambda_{\beta_{k,0}}\mathbf{1} \le \mathbf{1},\ 1 \le k \le K \\ & \mathbf{T}_k.\mathbf{1} = \mathbf{1}, \quad 1 \le k \le K \\ & \mathbf{t}_{k,i}^T \mathbf{m}_{k,i} = 0, \quad 1 \le i \le n, \quad 1 \le k \le K. \end{array} \tag{3}$$

2: Next, optimize $\mathbf{B}$ as:

$$\begin{array}{rl} \mathbf{B}^* = & \arg\min\limits_{\mathbf{B}} \sum_{k=1}^{K} \|\mathbf{y}_k - \lambda_{\alpha_{k,0}} \mathbf{D}\mathbf{B}\mathbf{T}_k^*\mathbf{x}_0 - \lambda_{\beta_{k,0}}\mathbf{1}\|_p \\ \text{subject to} & 0 \le \lambda_{\alpha_{k,0}} \mathbf{D}\mathbf{B}\mathbf{T}_k^*\mathbf{x}_0 + \lambda_{\beta_{k,0}}\mathbf{1} \le \mathbf{1},\ 1 \le k \le K \\ & \mathbf{B}.\mathbf{1} = \mathbf{1} \\ & \mathbf{b}_i^T \mathbf{e}_i = 0, \quad 1 \le i \le n. \end{array} \tag{4}$$

3: Next, optimize $\boldsymbol{\lambda}$ as:

$$\begin{array}{rl} \boldsymbol{\lambda}^* = & \arg\min\limits_{\boldsymbol{\lambda}} \sum_{k=1}^{K} \|\mathbf{y}_k - \lambda_{\alpha_k} \mathbf{D}\mathbf{B}^*\mathbf{T}_k^*\mathbf{x}_0 - \lambda_{\beta_k}\mathbf{1}\|_p \\ \text{subject to} & 0 \le \lambda_{\alpha_k} \mathbf{D}\mathbf{B}^*\mathbf{T}_k^*\mathbf{x}_0 + \lambda_{\beta_k}\mathbf{1} \le \mathbf{1},\ 1 \le k \le K \\ & \lambda_{\alpha_k} > 0, \quad 1 \le k \le K. \end{array} \tag{5}$$

4: Next, optimize $\mathbf{x}$ as:

$$\begin{array}{rl} \mathbf{x}^* = & \arg\min\limits_{\mathbf{x}} \sum_{k=1}^{K} \|\mathbf{y}_k - \lambda_{\alpha_k}^* \mathbf{D}\mathbf{B}^*\mathbf{T}_k^*\mathbf{x} - \lambda_{\beta_k}^*\mathbf{1}\|_p + \gamma\rho(\mathbf{x}) \\ \text{subject to} & 0 \ \le \ \mathbf{x} \ \le \ \mathbf{1} \\ & 0 \le \lambda_{\alpha_k}^* \mathbf{D}\mathbf{B}^*\mathbf{T}_k^*\mathbf{x} + \lambda_{\beta_k}^*\mathbf{1} \le \mathbf{1},\ 1 \le k \le K. \end{array} \tag{6}$$

5: Replace estimates in step 1 with $\mathbf{x}^*$, $\{\mathbf{T}_k\}^*$, $\mathbf{B}^*$ and $\boldsymbol{\lambda}^*$.

6: Repeat steps 1-5 till convergence.

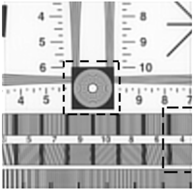
- Convex optimization problem in each individual variable
- Alternating minimization framework
- Dynamically evolving convex constraint:

$$\mathbf{0} \le \lambda_{\alpha_k}\mathbf{D}\mathbf{B}\mathbf{T}_k\mathbf{x} + \lambda_{\beta_k}\mathbf{1} \le \mathbf{1}, \quad 1 \le k \le K$$

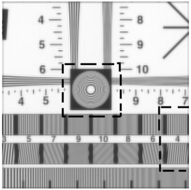
- $\{\mathbf{T}_k\}, \mathbf{B}, \boldsymbol{\lambda}, \mathbf{x}$ updated successively after steps 1, 2,3 and 4 respectively.
- Constraint still remains convex in the variable of interest.

Resolution enhancement: practical upper bound

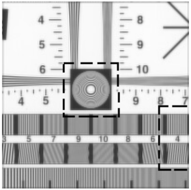
(a)



(b)



(c)



(d)

Figure: l_2 norm optimization: (a) Bilinearly interpolated lo-res image, (b) Result from Gunturk et al. (2006), (c) Practical upper bound: reconstructed from true imaging model parameters, (d) Result of proposed algorithm.

Joint resolution and dynamic range enhancement

(a)

(b)

(c)

Figure: Images courtesy Prof. Gunturk, LSU. (a) Four sample lo-res and LDR images, (b) Result from Gunturk et al. (2006), (c) Result from proposed algorithm.

Conclusions

- CACO framework unifies existing spatial domain SR techniques
- Improved algorithmic tractability
- Explicit optimization of $\{\mathbf{T}_k\}$ and $\mathbf{B}$, subject to physically meaningful convex constraints
- Remark: For l_2 norm, optimizing $\mathbf{B}$ and $\mathbf{T}_k$ can be shown to reduce to QP $\rightarrow$ efficient solvers.

Formulation as a QP

$$\arg\min_{\mathbf{W}} \|\mathbf{y} - \mathbf{W}\mathbf{x}\|^2 = \arg\min_{\mathbf{W}} \left((\mathbf{y} - \mathbf{W}\mathbf{x})^T(\mathbf{y} - \mathbf{W}\mathbf{x})\right)$$
$$= \arg\min_{\mathbf{W}} \left(\mathbf{x}^T\mathbf{W}^T\mathbf{W}\mathbf{x} - 2\mathbf{y}^T\mathbf{W}\mathbf{x}\right)$$

Let $\mathbf{Y} := \mathbf{x}\mathbf{x}^T$ and $\mathbf{w}_i^T$ denote the i-th row of $\mathbf{W}$. Then,

$$\begin{aligned} \mathbf{x}^T\mathbf{W}^T\mathbf{W}\mathbf{x} &= tr(\mathbf{x}^T\mathbf{W}^T\mathbf{W}\mathbf{x}) = tr(\mathbf{W}\mathbf{x}\mathbf{x}^T\mathbf{W}^T) \\ &= tr(\mathbf{W}\mathbf{Y}\mathbf{W}^T) = \sum_{i=1}^{K} \mathbf{w}_i^T\mathbf{Y}\mathbf{w}_i \\ &= vec(\mathbf{W}^T)^T(\mathbf{I}_m \otimes \mathbf{Y})vec(\mathbf{W}^T) \\ &= vec(\mathbf{W}^T)^T\tilde{\mathbf{Y}}vec(\mathbf{W}^T), \end{aligned}$$

where $vec(\cdot)$ is the vectorizing operator, $\mathbf{I}_m$ is the $m \times m$ identity matrix, and $\otimes$ represents the Kronecker product.

Formulation as a QP (contd.)

Similarly,

$$\begin{aligned}\mathbf{y}^T\mathbf{W}\mathbf{x} &= tr(\mathbf{y}^T\mathbf{W}\mathbf{x}) = tr(\mathbf{W}\mathbf{x}\mathbf{y}^T) \\ &= tr((\mathbf{x}\mathbf{y}^T)^T\mathbf{W}^T) = vec(\mathbf{x}\mathbf{y}^T)^T\ vec(\mathbf{W}^T).\end{aligned}$$

With $\mathbf{z} := vec(\mathbf{W}^T)$ and $\mathbf{c} := -2vec(\mathbf{x}\mathbf{y}^T)$, the original cost function becomes

$$\arg\min_{\mathbf{z}} \mathbf{z}^T\tilde{\mathbf{Y}}\mathbf{z} + \mathbf{c}^T\mathbf{z}. \qquad (1)$$

- $\mathbf{Y}$ positive semidefinite, $\mathbf{I}_m$ positive definite $\Rightarrow$ $\tilde{\mathbf{Y}}$ is positive semidefinite (Kronecker product preserves positive definiteness).
- Cost function is quadratic in $\mathbf{z}$.

What about the constraints?

- Membership constraint: Let $\mathbf{0} \in \mathbb{R}^m$ denote the vector with all zeros. Define $\mathbf{E} \in \mathbb{R}^{n \times mn}$ such that

$$\mathbf{E} = \begin{bmatrix} \mathbf{e}_1^T & \mathbf{0}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{e}_2^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T & \mathbf{e}_n^T \end{bmatrix} \tag{2}$$

$$\mathbf{w}_i^T \mathbf{e}_i = 0, i = 1, 2, \ldots, n \Leftrightarrow \mathbf{E}\mathbf{z} = \mathbf{0} (\in \mathbb{R}^n). \tag{3}$$

- Non-negativity:

$$w_{i,j} \geq 0 \Leftrightarrow \mathbf{z} \succeq \mathbf{0} (\in \mathbb{R}^{mn}). \tag{4}$$

- Interpolation constraint: Let $\mathbf{1} \in \mathbb{R}^m$ denote the vector with all ones. Define $\mathbf{F} \in \mathbb{R}^{n \times mn}$ such that

$$\mathbf{F} = \begin{bmatrix} \mathbf{1}^T & \mathbf{0}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{1}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T & \mathbf{1}^T \end{bmatrix} \tag{5}$$

$$\mathbf{W}.\mathbf{1} \ = \ \mathbf{1} \Leftrightarrow \mathbf{F}\mathbf{z} \ = \mathbf{1} (\in \mathbb{R}^n). \tag{6}$$

Complete optimization problem

$$\begin{aligned} \text{minimize} \quad & \mathbf{z}^T \tilde{\mathbf{Y}} \mathbf{z} + \mathbf{c}^T \mathbf{z} \\ \text{subject to} \quad & \mathbf{E}\mathbf{z} = \mathbf{0} \\ & \mathbf{z} \succeq \mathbf{0} \\ & \mathbf{F}\mathbf{z} = \mathbf{1} \end{aligned} \tag{7}$$

where

- $\mathbf{z} = vec(\mathbf{W}^T)$
- $\tilde{\mathbf{Y}} = \mathbf{I}_m \otimes (\mathbf{x}\mathbf{x}^T)$
- $\mathbf{c} = -2vec(\mathbf{x}\mathbf{x}^T)$.