Image-adaptive Color Super-resolution

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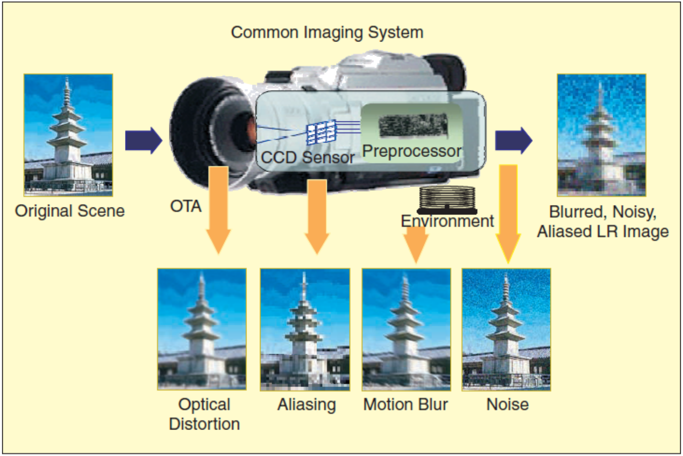
Color and Imaging Conference 2011

November 10, 2011

Outline

1. Gray-scale super-resolution (SR): State-of-the-art
2. Color SR: Background and motivation
3. Contribution: Image-adaptive color super-resolution
4. Results

Digital image acquisition system[1]



[1] Park et al., IEEE Signal Process. Mag., 2003

Model of the forward imaging process

$$\mathbf{y}_k = \mathbf{DBT}(\boldsymbol{\theta}_k)\mathbf{x} + \mathbf{n}_k, \quad 1 \le k \le K$$

- $\mathbf{x} \in \mathbb{R}^n \rightarrow$ unknown hi-res image
- $\mathbf{y}_k \in \mathbb{R}^m \ (m < n) \rightarrow k$-th lo-res image
- $\mathbf{T}(\boldsymbol{\theta}_k) \in \mathbb{R}^{n \times n} \rightarrow k$-th geometric warping matrix
  - $\boldsymbol{\theta}_k$ obtained from projective homography matrix[2]
- $\mathbf{B} \in \mathbb{R}^{n \times n} \rightarrow$ camera optical blur
- $\mathbf{D} \in \mathbb{R}^{m \times n} \rightarrow$ downsampling matrix of $1s$ and $0s$
- $\mathbf{n}_k \in \mathbb{R}^m \rightarrow$ noise vector that corrupts $\mathbf{y}_k$.

[2] Mann and Picard, IEEE Trans. Image Process., 1997

Prior work and key research challenges

$$\mathcal{C}(\mathbf{x},\boldsymbol{\theta}) = \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{DBT}(\boldsymbol{\theta}_k)\mathbf{x}\|_p, p \geq 1$$

$$(\hat{\mathbf{x}},\hat{\boldsymbol{\theta}}) = \arg\min_{\mathbf{x},\boldsymbol{\theta}} \mathcal{C}$$

1. Sequential estimation of $\{\theta_k\}$ and hi-res image $\mathbf{x}$
   - Sub-optimal
2. Cost function minimization under different norms[3] $\rightarrow$ different noise models
3. Joint MAP estimation[4] of $\{\theta_k\}$ and hi-res image $\mathbf{x}$
   - Tractability of optimization problem
   - Faithfulness of resulting solutions to real-world constraints.

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[4] Hardie et. al., IEEE Trans. Image Process., 1997

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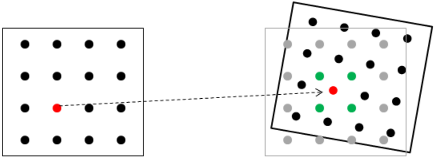
Addressing the challenges[5]

1. Separable convexity via transformation of variables $\mathbf{f}_k : \boldsymbol{\theta}_k \mapsto \mathbf{T}(\boldsymbol{\theta}_k)$
   - $\boldsymbol{\theta}$: change in pixel coordinates, $\mathbf{T}$: pixel intensity mapping



$$\mathcal{C}(\mathbf{x}, \{\mathbf{T}_k\}, \mathbf{B}) = \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{DBT}_k\mathbf{x}\|_p + \lambda\rho(\mathbf{x}).$$

2. Formulation of elegant and physically meaningful convex constraints.

Why convexity?

- Convergence guarantee to minima
- Robustness to initialization values.

[5] Monga and Srinivas, IEEE Asilomar Conf., 2010

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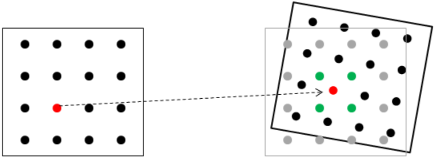

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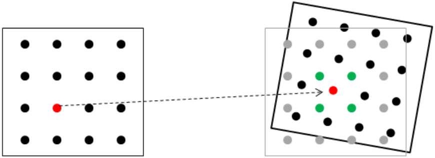

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Single-channel SR: Optimization problem

$$\underset{\mathbf{x},\{\mathbf{T}_k\},\mathbf{B}}{\text{minimize}} \quad \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{D}\mathbf{B}\mathbf{T}_k\mathbf{x}\|_p$$

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\text{subject to} \quad & \mathbf{0} \le \mathbf{x} \le \mathbf{1} \\
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- Non-negative pixel values of hi-res and lo-res images
- $\mathbf{T}_k$: interpolation matrix, $\mathbf{B}$: filtering with a local spatial kernel; each row should sum to 1
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Color super-resolution: Prior work

1. Treat RGB as independent channels → no channel correlation
2. Operate in a de-correlated color space[6]
   - Assumption: luminance component of image carries its spatial features
   - Chrominance components used mainly to improve image registration[7,8]
3. Strong correlation among spatial high-frequency components across color channels[9]
   - Related work: color image demosaicking[10].

[6] Vandewalle et al., Electronic Imaging, 2007
[7] Shah and Zakhor, IEEE Trans. Image Process., 1999
[8] Tom and Katsaggelos, IEEE Trans. Image Process., 2001
[9] Farsiu et al, IEEE Trans. Image Process., 2006
[10] Menon and Calvagno, IEEE Trans. Image Process., 2009

Luminance regularization

- $\mathbf{S}_r, \mathbf{S}_g, \mathbf{S}_b \in \mathbb{R}^{3n \times 3n}$: gradient operators on red, green and blue color channels respectively
- Luminance regularization (for images with dominant luminance edges):

$$\rho_L(\mathbf{x}) = \|(\mathbf{S}_r - \mathbf{S}_g)\mathbf{x}\|_1 + \|(\mathbf{S}_g - \mathbf{S}_b)\mathbf{x}\|_1 + \|(\mathbf{S}_r - \mathbf{S}_b)\mathbf{x}\|_1 \leq \epsilon_L.$$

- Modified optimization cost function:

$$\mathcal{C}_1 = \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{DBT}_k\mathbf{x}\|_p + \alpha_L \rho_L(\mathbf{x}).$$

- Successful for color SR $\rightarrow$ most images possess dominant luminance geometry.

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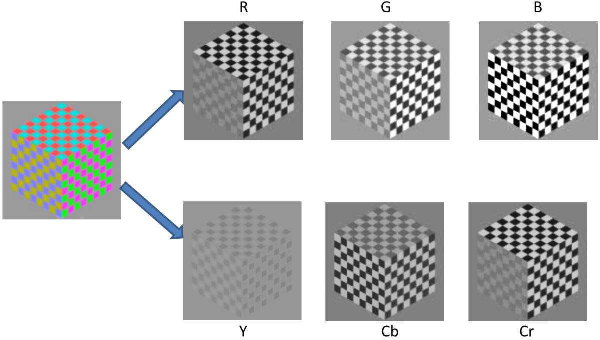
Motivation: Value of chrominance geometry



- Luminance edge (in Y) $\rightarrow$ present in R, G and B channels
- Chrominance edge $\rightarrow$ R, G and B channels with different high-frequency components
  - Strong edge in Cb $\rightarrow$ strong edge in B, mild edges in R and G.

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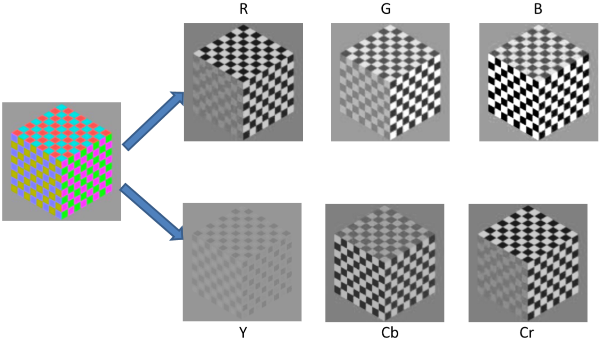

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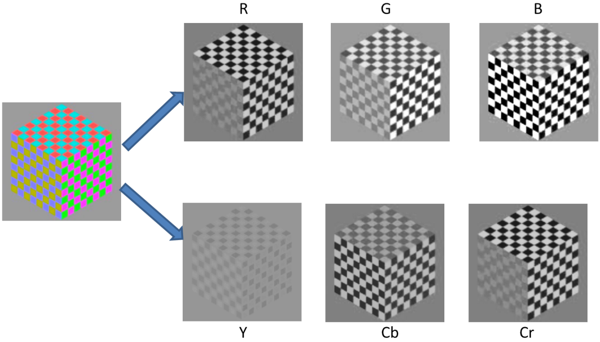

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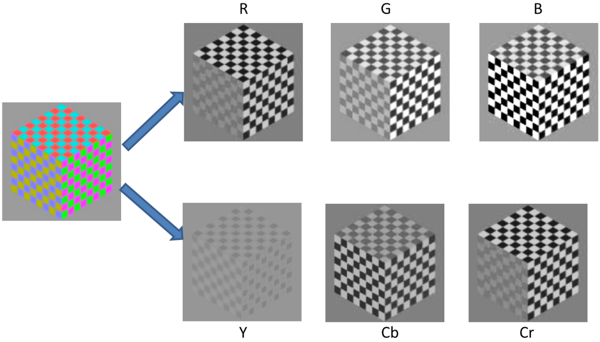

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Chrominance regularization: Intuition

- For images with significant chrominance geometry, edge correlation between RGB channels expected to be low

- Minimize edge correlation between channels in desired HR image:

$$(\mathbf{S}_r\mathbf{x})^T(\mathbf{S}_g\mathbf{x}) < \epsilon_{rg}, (\mathbf{S}_g\mathbf{x})^T(\mathbf{S}_b\mathbf{x}) < \epsilon_{gb}, (\mathbf{S}_b\mathbf{x})^T(\mathbf{S}_r\mathbf{x}) < \epsilon_{br}.$$

- Incorporate into cost function as a regularization term:

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Role of regularization: Venn diagram interpretation

$$\mathcal{C} = \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{DBT}_k\mathbf{x}\|_p + \alpha_L \rho_L(\mathbf{x}) + \alpha_C \rho_C(\mathbf{x}).$$

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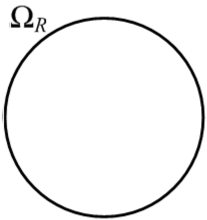

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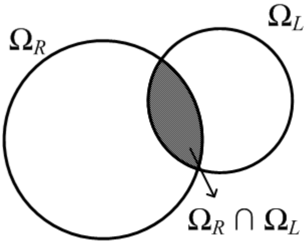

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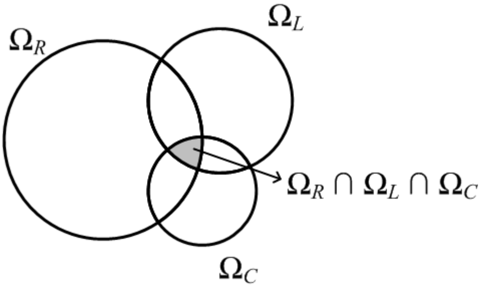

Contribution: Image-adaptive color super-resolution

$$\underset{\mathbf{x},\{\mathbf{T}_k\},\mathbf{B},\mathbf{S}_r,\mathbf{S}_g,\mathbf{S}_b}{\text{minimize}} \quad \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{D}\mathbf{B}\mathbf{T}_k\mathbf{x}\|_p + \alpha_L \rho_L(\mathbf{x}) + \alpha_C \rho_C(\mathbf{x})$$

subject to

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Constraints on gradient operators

- Gradient operator → high-pass filter ⇒ elements in each row must sum to zero:

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- $\mathbf{f}_{r,i}, \mathbf{f}_{g,i}, \mathbf{f}_{b,i}$ generated from initial gradient operator
- Element in $\mathbf{S}$ takes positive, negative or zero value → corresponding entry in $\mathbf{f}$ equals 1, -1 or 0 respectively.

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$$(\mathbf{s}_{r,i})^T \mathbf{f}_{r,i} = 1, (\mathbf{s}_{g,i})^T \mathbf{f}_{g,i} = 1, (\mathbf{s}_{b,i})^T \mathbf{f}_{b,i} = 1, 1 \leq i \leq 3n.$$

- $\mathbf{f}_{r,i}, \mathbf{f}_{g,i}, \mathbf{f}_{b,i}$ generated from initial gradient operator
- Element in $\mathbf{S}$ takes positive, negative or zero value $\rightarrow$ corresponding entry in $\mathbf{f}$ equals 1, -1 or 0 respectively.

How to choose α_C and α_L?

- Estimate degree of image chrominance geometry

$$\beta = \frac{1}{2}\left(\frac{\|\mathbf{H}_1\mathbf{x}_{\mathsf{Cb}}\| + \|\mathbf{H}_1\mathbf{x}_{\mathsf{Cr}}\|}{\|\mathbf{H}_1\mathbf{x}_{\mathsf{Y}}\|} + \frac{\|\mathbf{H}_2\mathbf{x}_{\mathsf{Cb}}\| + \|\mathbf{H}_2\mathbf{x}_{\mathsf{Cr}}\|}{\|\mathbf{H}_2\mathbf{x}_{\mathsf{Y}}\|}\right).$$

$$\mathbf{h}_1 = \begin{pmatrix} 3 & 10 & 3 \\ 0 & 0 & 0 \\ -3 & -10 & -3 \end{pmatrix},\ \mathbf{h}_2 = \begin{pmatrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{pmatrix}.$$

- $\beta \downarrow \Rightarrow \alpha_C \downarrow, \alpha_L \uparrow$
- $\beta \uparrow \Rightarrow \alpha_C \uparrow, \alpha_L \downarrow$
- α_C and α_L assigned complementary weights: $\alpha_C = \alpha_{\mathsf{max}} - \alpha_L$.

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Choice of β and α_C


(a) 0.584


(b) 0.864


(c) 0.294


(d) 0.483


(e) 0.497


(f) 0.503


(g) 0.425


(h) 0.828


(i) 0.975

Figure: Threshold: $\beta_0 = 0.75$.

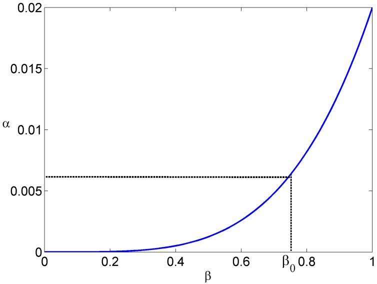
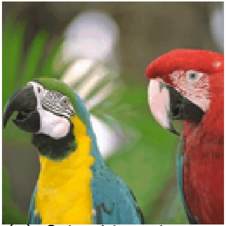
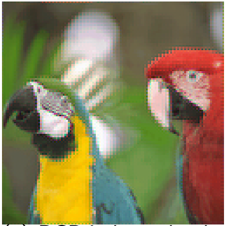


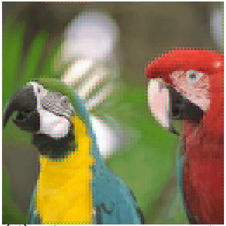
Figure: Mapping from β to α_c.

Results: I

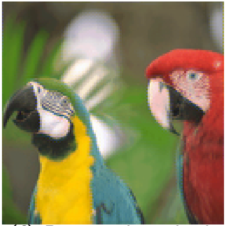

(a) Orig. hi-res image.


(b) Sample lo-res images.


(c) RGB independently.


(d) Luminance only (Farsiu).


(e) Lum. and chrom. independently (Vandewalle).


(f) Proposed method.

Results: II

(a) Interp. lo-res image.

(b) l_1-norm ρ_L(Farsiu)

(c) l_2-norm ρ_L(Menon)

(d) Lum. and chrom. independently (Vandewalle)

(e) Proposed framework (non-adaptive).

(f) Image-adaptive framework.

Conclusions

1. Color super-resolution framework that simultaneously exploits spatial and amplitude information
   - Novel chrominance regularization
   - Image-adaptive selection of optimization parameters
2. Constrained convex optimization framework
   - Tractable algorithms.

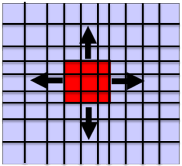
Thank you

Questions?

Backup Slides

High-pass filter constraints for gradient operators

$$H(\omega_1, \omega_2) = \sum_{(n_1,n_2)\in\mathcal{R}}\sum h[n_1, n_2]e^{-j\omega_1 n_1}e^{-j\omega_2 n_2}.$$



$$** \begin{bmatrix} h_{-1,-1} & h_{-1,0} & h_{-1,1} \\ h_{0,-1} & h_{0,0} & h_{0,1} \\ h_{1,-1} & h_{1,0} & h_{1,1} \end{bmatrix}$$

$$\begin{bmatrix} h_{-1,-1,} h_{0,-1,} h_{1,-1} \;\dots\; 0\;0\;0 \;\dots\; h_{-1,0,} h_{0,0,} h_{1,0} \;\dots\; 0\;0\;0 \;\dots\; h_{-1,1,} h_{0,1,} h_{1,1} \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

Quantitative comparison of performance

$$J_i = 10\left[\log\left(\frac{\sum_{k=1}^{K}\|\mathbf{y}-\mathbf{DBT}_k\mathbf{x}_i\|_2}{\sum_{k=1}^{K}\|\mathbf{y}-\mathbf{DBT}_k\mathbf{x}\|_2}\right) + (1-\beta)\log\frac{\rho_L(\mathbf{x}_i)}{\rho_L(\mathbf{x})} + \beta\log\frac{\rho_C(\mathbf{x}_i)}{\rho_C(\mathbf{x})}\right]$$

- Comparison with three competitive methods:
  1. $i = 1 : \alpha_L$ with l_1-norm[11]
  2. $i = 2 : \alpha_L$ with l_2-norm[12]
  3. $i = 3$: luminance and chrominance separately[13]
- $J > 0 \Rightarrow$ dB gain using proposed approach; $J < 0 \Rightarrow$ competitive method better.

Image	J_1	J_2	J_3
(a)	0.182	1.599	16.469
(b)	**7.900**	**6.562**	**26.856**
(c)	-0.493	-1.153	17.711
(d)	0.404	-0.979	12.196
(e)	7.902	4.674	21.647
(f)	7.222	5.260	20.804
(g)	9.806	8.388	21.588
(h)	**7.857**	**6.208**	**25.863**
(i)	**12.110**	**10.899**	**27.805**

[11] Farsiu et al., IEEE Trans. Image Process., 2006

[12] Menon et al., IEEE Trans. Image Process., 2009

[13] Vandewalle et al., SPIE 2007

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